STOCHASTIC VACUUM MODEL AND DUAL THEORY: A COMPARISON IN THE CONTEXT OF THE HEAVY QUARKONIA POTENTIAL

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The $Q\bar{Q}$ semirelativistic interaction in QCD can be expressed in terms of the Wilson loop and its functional derivatives. In this framework we discuss the complete (velocity and spin dependent) $1/m^2$ potential in the stochastic vacuum model and in the dual theory.

1 The Quark-Antiquark Potential

Up to order $1/m^2$ the quark-antiquark potential can be written as [1]:

$$\int_{t_\mathrm{i}}^{t_\mathrm{f}} dt V_{\mathrm{Q\bar{Q}}} = i \log\langle W(\Gamma)\rangle - \sum_{j=1}^{2} \frac{g}{m_j} \int_{\Gamma_j} dx^\mu \left(S_j^l \, \langle\!\langle \hat{F}_{l\mu}(x)\rangle\!\rangle \right.$$
$$\left. -\frac{1}{2m_j} S_j^l \varepsilon^{lkr} p_j^k \, \langle\!\langle F_{\mu r}(x)\rangle\!\rangle - \frac{1}{8m_j} \langle\!\langle D^\nu F_{\nu\mu}(x)\rangle\!\rangle \right) - \frac{1}{2} \sum_{j,j'=1}^{2} \frac{ig^2}{m_j m_{j'}}$$
$$\times \mathrm{T_s} \int_{\Gamma_j} dx^\mu \int_{\Gamma_{j'}} dx'^\sigma \, S_j^l \, S_{j'}^k \left(\langle\!\langle \hat{F}_{l\mu}(x) \hat{F}_{k\sigma}(x')\rangle\!\rangle - \langle\!\langle \hat{F}_{l\mu}(x)\rangle\!\rangle \, \langle\!\langle \hat{F}_{k\sigma}(x')\rangle\!\rangle \right) \quad (1)$$

where $W(\Gamma)$ is the Wilson loop over the closed path Γ including the quark and antiquark trajectories Γ_1 and Γ_2. The double bracket means the average on the gauge fields in presence of the Wilson loop. A path ordering is understood where ambiguities in the ordering of the colour matrices can arise.

The $1/m^2$ order terms in 1 are of two types, velocity dependent V_{VD} and spin dependent V_{SD}. Therefore we can identify in the full potential three types of contributions:

$$V_{\mathrm{Q\bar{Q}}} = V_0 + V_{\mathrm{VD}} + V_{\mathrm{SD}} \,, \quad (2)$$

where V_0 is the static potential. The spin independent part of the potential, $V_0 + V_{\mathrm{VD}}$, is obtained in 1 from the zero order and the quadratic terms in

the expansion of $\log\langle W(\Gamma)\rangle$ for small velocities. The terms of $V_{\rm VD}$ can be rearranged as [1]:

$$V_{\rm VD}(\mathbf{r}(t)) = \frac{1}{m_1 m_2}\left\{\mathbf{p}_1\cdot\mathbf{p}_2 V_{\rm b}(r) + \left(\frac{1}{3}\mathbf{p}_1\cdot\mathbf{p}_2 - \frac{\mathbf{p}_1\cdot\mathbf{r}\;\mathbf{p}_2\cdot\mathbf{r}}{r^2}\right)V_{\rm c}(r)\right\}_{\rm Weyl}$$
$$+\sum_{j=1}^{2}\frac{1}{m_j^2}\left\{p_j^2 V_{\rm d}(r) + \left(\frac{1}{3}p_j^2 - \frac{\mathbf{p}_j\cdot\mathbf{r}\;\mathbf{p}_j\cdot\mathbf{r}}{r^2}\right)V_{\rm e}(r)\right\}_{\rm Weyl}. \tag{3}$$

While the spin dependent potential $V_{\rm SD}$ can be rearranged as

$$\begin{aligned}V_{\rm SD} &= \frac{1}{8}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}\right)\Delta\left[V_0(r)+V_{\rm a}(r)\right]\\
&+\left(\frac{1}{2m_1^2}\mathbf{L}_1\cdot\mathbf{S}_1 - \frac{1}{2m_2^2}\mathbf{L}_2\cdot\mathbf{S}_2\right)\frac{1}{r}\frac{d}{dr}\left[V_0(r)+2V_1(r)\right]\\
&+\frac{1}{m_1m_2}\left(\mathbf{L}_1\cdot\mathbf{S}_2 - \mathbf{L}_2\cdot\mathbf{S}_1\right)\frac{1}{r}\frac{d}{dr}V_2(r)\\
&+\frac{1}{m_1m_2}\left(\frac{\mathbf{S}_1\cdot\mathbf{r}\,\mathbf{S}_2\cdot\mathbf{r}}{r^2}-\frac{1}{3}\mathbf{S}_1\cdot\mathbf{S}_2\right)V_3(r)+\frac{1}{3m_1m_2}\mathbf{S}_1\cdot\mathbf{S}_2\,V_4(r)\,,\end{aligned}\tag{4}$$

with $\mathbf{L}_j = \mathbf{r}\times\mathbf{p}_j$. All these potentials satisfy some identities following from the Lorentz invariance properties of the Wilson loop [2]:

$$\frac{d}{dr}\left[V_0(r)+V_1(r)-V_2(r)\right] = 0\,, \tag{5}$$
$$V_{\rm d}(r)+\frac{1}{2}V_{\rm b}(r)+\frac{1}{4}V_0(r)-\frac{r}{12}\frac{dV_0(r)}{dr} = 0\,, \tag{6}$$
$$V_{\rm e}(r)+\frac{1}{2}V_{\rm c}(r)+\frac{r}{4}\frac{dV_0(r)}{dr} = 0\,. \tag{7}$$

The expectation values in 1 can be expressed as functional derivatives of $\log\langle W(\Gamma)\rangle$ with respect to the quark trajectories $\mathbf{z}_1(t)$ or $\mathbf{z}_2(t)$. In particular

$$\langle\langle F_{\mu\nu}(z_j)\rangle\rangle = (-1)^{j+1}\frac{\delta i\log\langle W(\Gamma)\rangle}{g\,\delta S^{\mu\nu}(z_j)}\,, \tag{8}$$
$$\langle\langle F_{\mu\nu}(z_1)F_{\lambda\rho}(z_2)\rangle\rangle - \langle\langle F_{\mu\nu}(z_1)\rangle\rangle\langle\langle F_{\lambda\rho}(z_2)\rangle\rangle = -i\frac{\delta}{g\,\delta S^{\lambda\rho}(z_2)}\langle\langle F_{\mu\nu}(z_1)\rangle\rangle\,, \tag{9}$$

with $\delta S^{\mu\nu}(z_j)\equiv(dz_j^\mu\delta z_j^\nu - dz_j^\nu\delta z_j^\mu)$. Therefore, to obtain the whole quark-antiquark potential no other assumptions are needed than the behaviour of $\langle W(\Gamma)\rangle$.

2 Stochastic Vacuum Model

Assuming the stochastic vacuum model behaviour of the Wilson loop [3] from 1, 8, 9 we obtain in the long-range limit [4,5]:

$$
\begin{aligned}
V_0(r) &= \sigma_2 r + \frac{1}{2}C_2^{(1)} - C_2\,, && (10)\\
V_{\rm b}(r) &= -\frac{1}{9}\sigma_2 r - \frac{2}{3}\frac{D_2}{r} + \frac{8}{3}\frac{E_2}{r^2}\,, && (11)\\
V_{\rm c}(r) &= -\frac{1}{6}\sigma_2 r - \frac{D_2}{r} + \frac{2}{3}\frac{E_2}{r^2}\,, && (12)\\
V_{\rm d}(r) &= -\frac{1}{9}\sigma_2 r + \frac{1}{4}C_2 - \frac{1}{8}C_2^{(1)} + \frac{1}{3}\frac{D_2}{r} - \frac{2}{9}\frac{E_2}{r^2}\,, && (13)\\
V_{\rm e}(r) &= -\frac{1}{6}\sigma_2 r + \frac{1}{2}\frac{D_2}{r} - \frac{1}{3}\frac{E_2}{r^2}\,, && (14)\\
\Delta V_{\rm a}(r) &= \text{ self-energy terms}\,, && (15)\\
\frac{d}{dr}V_1(r) &= -\sigma_2 + \frac{C_2}{r}\,, && (16)\\
\frac{d}{dr}V_2(r) &= \frac{C_2}{r}\,, && (17)\\
V_3(r) &= \text{ falls off exponentially in } r\,, && (18)\\
V_4(r) &= \text{ falls off exponentially in } r\,, && (19)
\end{aligned}
$$

where σ_2 is the string tension in the bilocal approximation, and C_2, $C_2^{(1)}$, D_2, E_2 are some constants, which can be fixed from the lattice data. At the leading order in the long-range limit, neglecting exponentially falling off terms, these results coincide with those obtained in the so-called minimal area law model [1] and reproduce the Buchmüller flux tube picture of the quark-antiquark interaction.

3 Dual Theory

Assuming duality, the behaviour of the Wilson loop follows from the classical configuration of dual potentials and monopoles in the dual theory [6]. Substituting in 1 we obtain in the long-range behaviour [4,5]:

$$
\begin{aligned}
V_0(r) &= \sigma r - 0.646\sqrt{\sigma\alpha_{\rm s}}\,, && (20)\\
\frac{d}{dr}V_1(r) &= -\sigma + \frac{0.681}{r}\sqrt{\sigma\alpha_{\rm s}}\,, && (21)\\
\frac{d}{dr}V_2(r) &= \frac{0.681}{r}\sqrt{\sigma\alpha_{\rm s}}\,, && (22)
\end{aligned}
$$

$$V_3(r) = \frac{4}{3}\alpha_{\rm s} \, \left(M^2 + \frac{3}{r} M + \frac{3}{r^2} \right) \, \frac{e^{-Mr}}{r} \, , \tag{23}$$

$$V_4(r) = \frac{4}{3}\alpha_{\rm s} \, M^2 \frac{e^{-Mr}}{r} \, , \tag{24}$$

$$V_{\rm b}(r) = -0.097 \, \sigma r - 0.226\sqrt{\sigma\alpha_{\rm s}} \, , \tag{25}$$

$$V_{\rm c}(r) = -0.146 \, \sigma r - 0.516\sqrt{\sigma\alpha_{\rm s}} \, , \tag{26}$$

$$V_{\rm d}(r) = -0.118 \, \sigma r + 0.275\sqrt{\sigma\alpha_{\rm s}} \, , \tag{27}$$

$$V_{\rm e}(r) = -0.177 \, \sigma r + 0.258\sqrt{\sigma\alpha_{\rm s}} \, , \tag{28}$$

where $M \approx 600$MeV is the dual gluon mass, $\sigma \approx 0.18$ GeV^2 is the string tension and $\alpha_{\rm s} \approx 0.39$. For an evaluation of $\Delta V_{\rm a}$ we refer the reader to [5]. The agreement between the results in the stochastic vacuum model and in the dual theory is good. In particular, we notice that both models reproduce the flux tube picture, show up the same $1/r$ correction to the dV_1/dr potential and non vanishing spin-spin potentials. For a more detailed discussion we refer the reader to [4,5]. All these potentials have been recently evaluated on the lattice [7], confirming, up to now, the theoretical predictions.

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